1. Find the greatest common factor of 22269929 and 5620813. 4961

2. Find the prime factorization of 16,724,261,299,239. $3^5 \cdot 13^3 \cdot 29^2 \cdot 193^2$

3. Find the smallest pair of twin primes greater than 2000. 2027 & 2029

4. Find the smallest pair of twin primes greater than 3000. 3251 & 3253

5. Find the next pair of twin primes greater than 3000 (i.e. the first pair greater than the pair found in the previous problem). 3257 & 3259

6. Find the prime factorization of 86583. $19 \cdot 31 \cdot 7^2 \cdot 3$

7. Find the prime factorization of 217951. $67 \cdot 3253$

8. Find the prime factorization of 24151. 24151

9. A balanced prime is a prime which is the average of the previous prime and the following prime. Find the first balanced prime greater than 200. 211

10. Find the next balanced prime greater than 200 (i.e. the next balanced prime after the one you found in the previous problem). 257

11. Find the greatest even prime less than 100. 2

12. A circular prime number is a number that remains prime on any cyclic rotation of its digits. For each of the following, circle the number if it is a circular prime.

[redacted]
b) 731
c) 829
[redacted]
e) 1153

13. Determine if 19801 is prime, and if so, find $n > 0$ so that 19801 is equal to one of the following forms, and then indicate the form.

a) Carol Primes: $\left(2^n - 1\right)^2 - 2$

b) Centered Decagonal Primes: $5\left(n^2 - n\right) + 1$

c) Centered Heptagonal Primes: $\frac{\left(7n^2 - 7n + 2\right)}{2}$

d) Centered Square Primes: $n^2 + (n+1)^2$

e) Centered Triangular Primes: $\frac{\left(3n^2 + 2n + 2\right)}{2}$

n = 99

Form: Centered Square Prime

14. Determine if 11173 is prime, and if so, find $n > 0$ so that 11173 is equal to one of the following forms, and then indicate the form.

a) Carol Primes: $\left(2^n - 1\right)^2 - 2$

b) Centered Decagonal Primes: $5\left(n^2 - n\right) + 1$

c) Centered Heptagonal Primes: $\frac{\left(7n^2 - 7n + 2\right)}{2}$

d) Centered Square Primes: $n^2 + (n+1)^2$

e) Centered Triangular Primes: $\frac{\left(3n^2 + 2n + 2\right)}{2}$

n = 57

Form: Centered Heptagonal Prime

15. If possible, determine how many real solutions $\sin\left(\frac{\pi}{x-1}\right) = 0$ has on each of the following intervals.

a) [0, 0.8] 5 (1 point)

b) [0, 0.9] 10 (1 point)

c) [0, 0.95] 20 (1 point)

d) [0, 1] Not possible to determine (2 points)

16. Solve x = 27

$$\cfrac{\cfrac{\cfrac{\cfrac{\cfrac{x!}{(2x+10)^2(x+5)^2}-x^5}{x^4}-\left(4x^3+3x^2+11x+7\right)}{7x^3-11x^2+9x-5}-\left(11x^2+8x-15\right)}{2x^2-4x+20}-\left(3x^3-16x^2+3x+50\right)}{16x^2+8x-1}-299=2010$$

17. Solve $(x^2-9x+19)^{(2x^3-x^2-10x)}=1$. x = -2, 2.5, 3, 4, 6

18. What is the sum of all the intercepts of the graph of $f(x)=25x^4-171x^2-154x-24$? 0

19. Find the exact value of $\sum_{k=1}^{\infty}\frac{k}{5^k}$. $\frac{5}{16}$

20. How many times does $y=15\sin(360x)$ take on the value 10 on the interval $[0,1]$? 115

21. $\sqrt{2+\sqrt{2+\sqrt{2+\ldots}}}$ = 2

22. How many pairs of integers (*a*, *b*) are there for which $a^2-b^2=256$.
14 $(a,b)\in\{(\pm16,0),(\pm20,\pm12),(\pm34,\pm30),(\pm65,\pm63)\}$

23. Create a graph in polar coordinates of a flower with 16 petals where each petal is 3 units long and no petal lies on an axis. Show the graph to an instructor. But first, be sure to set the graphing window and scales so that the tips of the petal look like the tips of petals (rounded) AND so that the instructor can readily see the lengths of the petals, the number of petals, and their placement with regard to the axes (bring your answer sheet with you).

It's a graph of $r=3\sin(8\varphi)$ with a φ-step of no more than $\frac{\pi}{96}$.

AREA OF A TRIANGLE:

The area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) can be determined by evaluating the following determinant:

$$Area = \pm\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

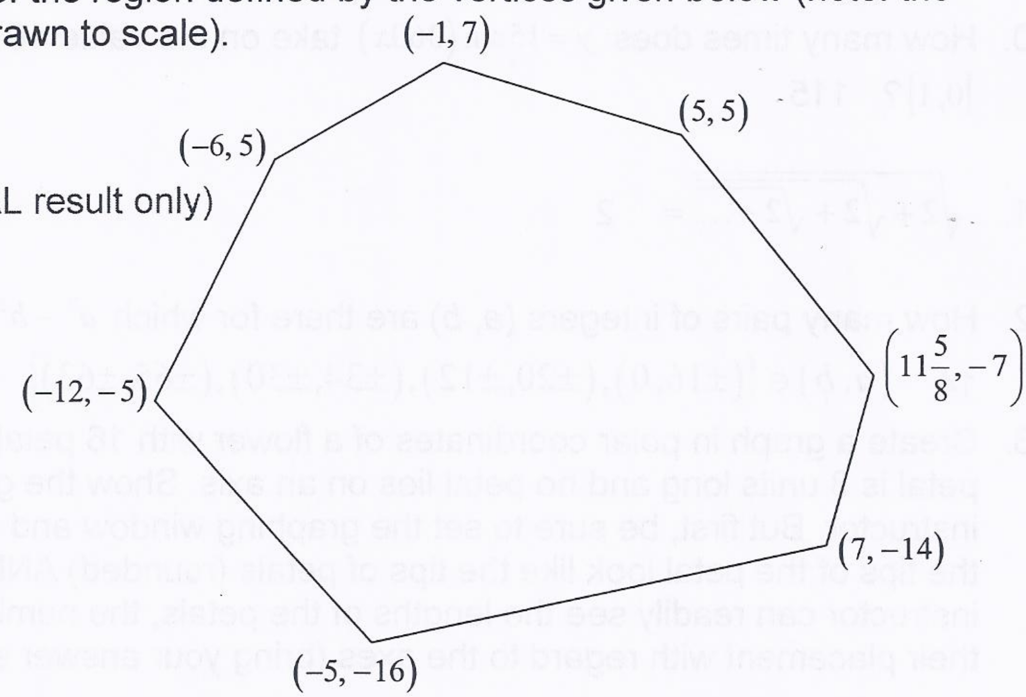
Where the $(\pm)$ symbol indicates that the appropriate sign should be chosen to yield a positive area. Find the areas of the following triangles, determined by the given points. Round final answers to the nearest tenth if necessary.

24. Triangle's vertices: $(-4, 6)$, $(1, 4)$, and $(14, -15)$. 34.5

25. Triangle's vertices: $(\ln(5), \sqrt{17})$, $(\ln(12), -32)$, and $(e^3, 10)$. 336.3

26. Find the area of the region defined by the vertices given below (note: the shape is not drawn to scale).

361.4
(rounding FINAL result only)



27. Find all values of y for which the triangle with vertices $(-4, 6)$, $(y, 1)$, and $(-4, -11)$ has an area of 68. y = 4 OR y = -12

STATISTICS:

Without using the variance and standard deviation functions of your graphing calculator, we'll find the variance and standard deviation for the following data observations (our method will yield slightly different results than the statistics functions on your calculator would produce).

2	3	1	5	4	8	6	1	1	2
7	5	6	8	9	1	10	9	6	6
5	0	1	2	4	4	7	7	8	3

28. Create a list, L1, of these data, and use the list to find the mean, $\bar{x}$, for these data (round to the nearest thousandth if necessary). 4.7

29. Create a second list, L2, of the deviations from the mean; i.e. calculate $x-\bar{x}$, for each observation, x, in L1 and store these deviations in L2. Now create a third list, L3, of the squares of the deviations in L2; i.e. calculate $(x-\bar{x})^2$ for each observation, x, and store in L3. Sum L3 (round to the nearest tenth if necessary). 240.3

30. To find the variance for the original data observations, find the arithmetic mean of the entries in L3. Note: our variance may differ from applying the graphing calculator's variance function to the original data observations. (Round to the nearest hundredth if necessary). 8.01

31. The standard deviation, *sd*, for the original data observations is the square root of the variance found in the previous problem. Find the standard deviation, *sd* (round to the nearest thousandth if necessary). 2.830

32. The z-score (or standard score) for any given observation, x, is $\frac{x-\bar{x}}{sd}$ (use the rounded *sd* found in the previous problem). Create a fourth list, L4, of the z-scores for the original data observations. What is the maximum z-score in the list (rounded to the nearest thousandth, if necessary). 1.873

ALGEBRA TOPICS:

Unless you are directed to do otherwise, round irrational final results to the nearest hundredth. Do not round rational final results.

33. Use your calculator to find the equation of the quadratic function that passes through the following points:

(-6, -58.5), (7, -7.15), and (21, 89.73) $y = 0.11x^2 + 3.84x - 39.42$

34. Use your calculator to find the equation of the cubic function that passes through the following points:

(-8, -164.8), (-2, 9.8), (6, 63.4), and (17, 1892.7) $y = 0.4x^3 - 4.5x + 4$

35. Use your calculator to find the equation of the exponential function passing through the points:

(-3, 6.25) and (2, 2.048) $y = 3.2(0.8)^x$

36. Solve the following system of equations (round to the nearest hundredth if necessary).

$$\begin{aligned} 3x - 2.8y &= 21.06 \\ 2x + 1.2y &= 5.3 \end{aligned}$$

(4.36, -2.85)

37. Solve the following system of equations (round to the nearest hundredth if necessary).

$$\begin{aligned} x + 2y - z &= -3 \\ \frac{1}{3}x - y + \frac{1}{3}z &= 2 \\ x + \frac{1}{2}y + z &= \frac{5}{2} \end{aligned}$$

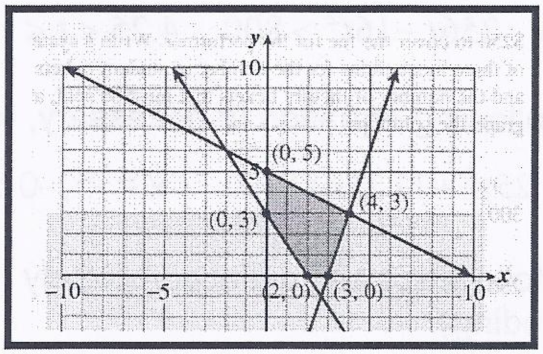
(1, -1, 2)

38. Find all points of intersection of the graphs of the given equations (round to the nearest hundredth if necessary).

$(x-3)^2+(y-2)^2=9$ (0.30, 3.30), (5.70, 3.30), (4.92, -0.30), (1.08, -0.30)
$(x-3)^2-y=4$

39. Use your calculator to graph the solution set to the following system of inequalities. Show the graph of the solution set to the instructors at the front of the room. Be sure to carefully select an appropriate graphing window, x-scale, and y-scale so that the instructor can readily determine all intercepts and intersections.

$$2y + 3x \geq 6$$
$$2y + x \leq 10$$
$$y \geq 3x - 9$$
$$x \geq 0 \quad , \quad y \geq 0$$



40. Solve the inequality, and write your solution in <u>inequality notation</u> (round to the nearest hundredth if necessary).

$x^2-3x-180>0$ $x<-12$ or $x>15$

41. Solve the inequality, and write your solution in <u>inequality notation</u> (round to the nearest hundredth if necessary).

$-6x^2+4.8x\geq 0.9$ $0.3\leq x\leq 0.5$

42. Solve the inequality, and write your solution in <u>interval notation</u> (round to the nearest hundredth if necessary).

$x^2-3x-18\geq -8$ $(-\infty,-2]\cup[5,\infty)$

43. The Chamber of Commerce in River City wants to put on a fourth of July fireworks display. A city ordinance requires that fireworks at public gatherings explode higher than 800 feet above the ground. The mayor particularly wants to include the Freedom Starburst model, which is launched from the ground so that it's height after t seconds is given by the following function.

$h = 256t - 16t^2$

Set up an inequality to determine when the starburst should explode to satisfy the city ordinance. Solve the inequality and write the inequality and the solution on your answer sheet (round to the nearest hundredth if necessary). $256t - 16t^2 > 800$, $4.25 < t < 11.75$

44. Solve the following equation (if necessary, round to the nearest hundredth).

$x^4 - 6x^3 + 3.25x^2 + 17.25x + 7 = 0$ x = -1, -0.5, 3.5, 4

45. Factor the following polynomial completely (use only integers – no fractions and no rounding).

$24x^5 - 86x^4 - 219x^3 + 199x^2 + 222x + 40$ (3x - 4)(2x + 1)(x - 5)(x + 2)(4x + 1)

Use the matrices defined below for the following four problems. Do not use decimal notation in your matrix entries. Use fraction notation for rational numbers (excluding integers).

$$A=\begin{bmatrix}-1 & 0 & 2\\ 3 & 1 & -4\\ -2 & -3 & 5\end{bmatrix} \quad B=\begin{bmatrix}-5 & 1 & 0\\ 2 & 0 & 3\\ -7 & 9 & -1\end{bmatrix}$$

46. Find AB. AB = $\begin{bmatrix}-9 & 17 & -2\\ 15 & -33 & 7\\ -31 & 43 & -14\end{bmatrix}$

47. If $f(x)=5x^4+2x^3-5x^2-2$, evaluate $f(A)$. f(A) = $\begin{bmatrix}-924 & -1050 & 1816\\ 2024 & 2292 & -3982\\ -3391 & -3774 & 6624\end{bmatrix}$

48. Find the multiplicative inverse of matrix B (remember to use fractions – not decimals).

$$B^{-1}=\begin{bmatrix}-27/116 & 1/116 & 3/116\\ -19/116 & 5/116 & 15/116\\ 9/58 & 19/58 & -1/58\end{bmatrix}$$

49. Find the matrix X such that $-2X+7A=4B$. (Remember to use fractions – not decimals).

$$\begin{bmatrix}13/2 & -2 & 7\\ 13/2 & 7/2 & -20\\ 7 & -57/2 & 39/2\end{bmatrix}$$

CRYPTOLOGY:

A cryptogram is a message written according to a secret code. Matrix multiplication can be used to encode and decode messages. First we assign a number to each letter of the alphabet (A = 1, B = 2, C = 3, …, Z = 26) and assign the number 0 to a blank space. Then any message is converted to numbers and partitioned into uncoded row matrices, each with the appropriate number of entries.

For example, if our encoding matrix is a 3X3 matrix, then we would partition the message "MEET ME MONDAY" into the following 1X3 row matrices.

[13 5 5] [20 0 13] [5 0 13] [15 14 4] [1 25 0]

Let's use $A=\begin{bmatrix}-5 & 1 & 2\\ 3 & 2 & -4\\ -2 & -1 & 5\end{bmatrix}$ to encode the first segment of the message.

Uncoded Message	Encoding Matrix A	Coded Message

$$\begin{bmatrix}13 & 5 & 5\end{bmatrix}\begin{bmatrix}-5 & 1 & 2\\ 3 & 2 & -4\\ -2 & -1 & 5\end{bmatrix}=\begin{bmatrix}-60 & 18 & 31\end{bmatrix}$$

$$\begin{bmatrix}20 & 0 & 13\end{bmatrix}\begin{bmatrix}-5 & 1 & 2\\ 3 & 2 & -4\\ -2 & -1 & 5\end{bmatrix}=\begin{bmatrix}-126 & 7 & 105\end{bmatrix}$$

$$\begin{bmatrix}5 & 0 & 13\end{bmatrix}\begin{bmatrix}-5 & 1 & 2\\ 3 & 2 & -4\\ -2 & -1 & 5\end{bmatrix}=\begin{bmatrix}-51 & -8 & 75\end{bmatrix}$$ and so on and so on …

And we would present the coded message as follows.

-60, 18, 31, -126, 7, 105, -51, -8, 75, and so on and so on …

50. Given the encoding matrix $A = \begin{bmatrix} -5 & 1 & 2 \\ 3 & 2 & -4 \\ -2 & -1 & 5 \end{bmatrix}$, encode the following message.

PLEASE SEND MONEY
-54, 35, 9, 42, 34, -49, 47, 33, -51, -58, 22, 12, -48, 29, 36, 50, 55, -90

51. Given that the encoding matrix $A = \begin{bmatrix} -5 & 1 & 2 \\ 3 & 2 & -4 \\ -2 & -1 & 5 \end{bmatrix}$ was used to create the following coded message, decode the message.

39, 32, -45, 59, 41, -67, -18, -7, 59, -55, 24, 8, -115, 15, 130
ARE WE DONE YET

52. (7 points) Draw a skate-boarder moving across your graphing window. Show it to one of the instructors (bring your answer sheet with you).

Partial credit if the skate-boarder just moves across the screen.
Full credit if the dude crashed or does a trick.